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Energetic Model of the Tire-Ground Interaction and Comparison with LuGre Friction Model

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Abstract
In this paper the energetic dynamic model of the tire-ground interaction in a vehicle is presented and a comparison with the LuGre friction model is given. The Power-Oriented Graphs (POG) technique is exploited for both representing and analyzing the two dynamic systems. In particular the LuGre model is analyzed showing some drawbacks and a modified model is proposed in order to have an always dissipative model. Simulation results end the paper comparing the two dynamic models of the tire-ground interaction.

Keywords: Vehicle dynamics, tire-road interaction, friction model.

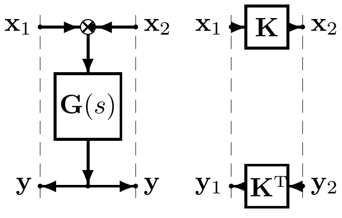
1. INTRODUCTION

Friction force at the tire-ground (vehicle-terrain) interface is the main mechanism to convert motor torque to longitudinal force. In the literature many models of the friction force have been proposed, such as Pacejka formulas [1], Dahl's model and LuGre model (see [2], [3] for a reviewing of the main friction models). The need of a model describing the interaction between the tire and the road has more and more increased, in particular there is the need of a model able to catch the main nonlinear behaviors of friction and at the same time not too complex and with a few parameters to be tuned. The traditional approach to this interaction involves Pacejka's formulas [1]. These formulas are based on empirical data fitting and have some limits to their applicability. In fact they are static functions involving a great number of parameters that have to be set in function of different conditions such as road surface, tire pressure, vehicle load etc. Moreover they always require the presence of a slip to generate forces, but in this way it is not possible, for example, to simulate a vehicle at rest on an inclined surface. The LuGre model, proposed in [4] and then revisited in [5], is a dynamic model able to describe also the stick-slip motion. In [6] the authors introduced an energetic dynamic model based on the elastic interaction of the tire with the ground.

In this paper the energetic model is presented considering only the longitudinal friction and the LuGre model is analyzed to give a comparison between the main properties of both models. The Power-Oriented Graphs technique is used for modelling the elastic interaction of the tire with the ground. The energetic approach can avoid some drawbacks of other models and moreover it has always dissipativity properties. The paper is organized in the following way. Section 2 states the main properties of the Power-Oriented Graphs tecnhique, Section 3 presents the energetic model of the tire-ground interaction, Section 4 is about LuGre model and the comparison between the two friction models, in Section 5 shows some simulation results and finally some conclusions are reported in Section 6.

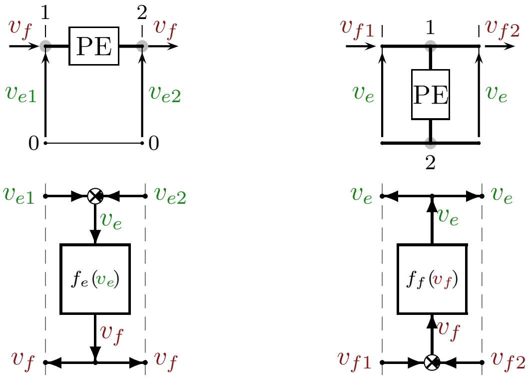
2. POWER-ORIENTED GRAPHS BASIC PRINCIPLES

The Power-Oriented Graphs technique, see [7] and [6], is suitable for modeling physical systems. The POG block schemes are normal block diagrams combined with a particular modular structure essentially based on the use of the two blocks shown in Fig. 1.a and Fig. 1.b: the *elaboration block* (e.b.) stores and/or dissipates energy (i.e. springs, masses, dampers, capacities, inductances, resistances, etc.); the *connection block* (c.b.) redistributes the power within the system without storing nor dissipating energy (i.e. any type of gear reduction, transformers, etc.). The e.b. and the c.b. are suitable for representing both scalar and vectorial systems. In the vectorial case, $\mathbf{G}(s)$ and $\mathbf{K}$ are matrices: $\mathbf{G}(s)$ is always a square matrix composed by positive real transfer functions; matrix $\mathbf{K}$ can also be rectangular. The circle present in the e.b. is a summation element and the black spot represents a minus sign that multiplies the entering variable. The main feature of the Power-Oriented Graphs is to keep a direct correspondence between the dashed sections of the graphs and real power sections of the modeled systems: the scalar product $\mathbf{x}^{\mathrm{T}}\mathbf{y}$ of the two *power vectors* $\mathbf{x}$ and $\mathbf{y}$ involved in each dashed line of a power-oriented graph, see Fig. 1, has the physical meaning of *the power flowing through that particular section.*



a) *e. b.* b) *c. b.*

Figure 1. POG basic blocks and variables: a) *elaboration block*, b) *connection block*.

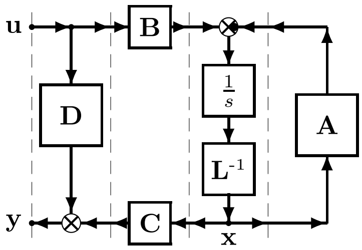


a) Element in series. b) Element in parallel.

Figure 2. POG representations of Physical Elements (PE): a) connected in series (inputs v_{e1}, v_{e2}); b) connected in parallel (inputs v_{f1}, v_{f2}).

The main energetic domains encountered in modeling physical systems are the electrical, the mechanical (translational and rotational) and the hydraulic. Each energetic domain is characterized by two *power variables*: an *across-variable* v_e defined between two points (i.e. the voltage V, the velocity $\dot{x}$, etc.), and a *through-variable* v_f defined in each point of the space (i.e. the current I, the force F, etc.). Each Physical Element (PE) interacts with the external world through the power sections associated to its terminals. There is a direct correspondence between physical power sections and dashed sections in the POG model. Each power variable must be defined with its positive reference direction of the power. When considering a single physical element it is made the assumption of power always entering through all the terminals of the element. Two physical elements can be connected by making one or both of their terminals coincide. When two physical elements are connected, a feedback always appears. A physical element can be connected to another element in only two ways: *in series* or *in parallel*. A Physical Element is connected *in series* when its terminals share the same through-variable v_f: see the physical element and the corresponding POG scheme in Fig. 2.a. A Physical Element is connected *in parallel* when its terminals share the same across-variable v_e: see the physical element and the POG scheme in Fig. 2.b. When two physical elements are connected in series or parallel, their corresponding mathematical models appear to be connected in feedback, whatever the orientation of each model is.

Another important property of the POG technique is the direct correspondence between the POG schemes and the corresponding state space dynamic equations. For example, the POG scheme shown in Fig. 3 can be represented by the state space equations (1) where the *energy matrix* $\mathbf{L}$ is symmetric and positive definite: $\mathbf{L} = \mathbf{L}^{\mathrm{T}} > 0$. It can be easily shown that when $\mathbf{D} = 0$ it follows that $\mathbf{C} = \mathbf{B}^{\mathrm{T}}$. When an eigenvalue of matrix $\mathbf{L}$ tends to zero (or to infinity), system (1) degenerates towards a lower dimension dynamic system. In this case, the dynamic model of the "reduced" system, see (2), can be directly obtained from (1) using a simple "congruent" transformation $\mathbf{x} = \mathbf{T}\,\mathbf{z}$ where (if $\mathbf{T}$ is constant) $\overline{\mathbf{L}} = \mathbf{T}^{\mathrm{T}}\mathbf{L}\mathbf{T}$, $\overline{\mathbf{A}} = \mathbf{T}^{\mathrm{T}}\mathbf{A}\mathbf{T}$, $\overline{\mathbf{B}} = \mathbf{T}^{\mathrm{T}}\mathbf{B}$, $\overline{\mathbf{C}} = \mathbf{C}\mathbf{T}$ and $\overline{\mathbf{D}} = \mathbf{D}$.



$$\begin{cases} \mathbf{L}\,\dot{\mathbf{x}} = -\mathbf{A}\,\mathbf{x} + \mathbf{B}\,\mathbf{u} \\ \mathbf{y} = \mathbf{C}\,\mathbf{x} + \mathbf{D}\,\mathbf{u} \end{cases} \tag{1}$$

$$\Downarrow (\mathbf{x} = \mathbf{T}\,\mathbf{z})$$

$$\begin{cases} \overline{\mathbf{L}}\,\dot{\mathbf{z}} = -\overline{\mathbf{A}}\,\mathbf{z} + \overline{\mathbf{B}}\,\mathbf{u} \\ \mathbf{y} = \overline{\mathbf{C}}\,\mathbf{z} + \overline{\mathbf{D}}\,\mathbf{u} \end{cases} \tag{2}$$

Figure 3. POG block scheme of a generic dynamic system.

The POG linear systems described in form

$$\overline{\mathbf{L}}\,\dot{\mathbf{z}} = -\overline{\mathbf{A}}\,\mathbf{z} + \overline{\mathbf{B}}\,\mathbf{u}, \qquad \mathbf{y} = \overline{\mathbf{B}}^{\mathrm{T}}\mathbf{z} \tag{3}$$

always satisfy the following properties:

1) the energy E_s stored in the system and the dissipating power P_d are quadratic functions of matrices $\overline{\mathbf{L}}$ and $\overline{\mathbf{A}}_s$, respectively:

$$E_s = \frac{1}{2}\,\mathbf{z}^{\mathrm{T}}\,\overline{\mathbf{L}}\,\mathbf{z}, \qquad P_d = \mathbf{z}^{\mathrm{T}}\,\overline{\mathbf{A}}_s\,\mathbf{z}$$

where $\overline{\mathbf{A}}_s = (\overline{\mathbf{A}} + \overline{\mathbf{A}}^{\mathrm{T}})/2$ is the symmetric part of the *power matrix* $\overline{\mathbf{A}}$. The skew-symmetric part $\overline{\mathbf{A}}_w = (\overline{\mathbf{A}} - \overline{\mathbf{A}}^{\mathrm{T}})/2$ of matrix $\overline{\mathbf{A}}$ represents the power redistribution within the system. One can easily verify that all the dissipating parameters of the system appear only in matrix $\overline{\mathbf{A}}_s$, while matrix $\overline{\mathbf{A}}_w$ is completely characterized by all the connection parameters;

2) all the loops of the POG schemes *always contain an "odd" number of signs "−"* (i.e. the black spots) in the summation blocks of the loop;

3) the direction of the power flowing through a section is positive if *an "even" number of signs "−" is present along all the paths* linking the input and output of the section.

3. ENERGETIC MODEL OF THE TIRE-GROUND INTERACTION

The traditional approach to the tire-ground interaction involves Pacejka's formulas [1] which are based on empirical data fitting and have some limits to their applicability. In fact they are static functions involving a great number of parameters that have to be set in function of different conditions such as road surface, tire pressure, vehicle load etc. They always require the presence of a slip $\lambda = \frac{\omega R - v_x}{v_x}$ to generate forces, but in this way it is not possible, for example, to simulate a vehicle at rest on an inclined surface because they provide zero forces and torques when the slip-ratio λ is zero. In this case, however, even if the slip-ratio is zero, a longitudinal force must be present to keep the vehicle stopped on the inclined surface. Moreover Pacejka's formulas mix together the "slip" and "skid" phenomena, in fact for small values of λ these formulas essentially describe the "slip" of the tire due to the rolling of the wheel when a force is generated at the ground, while for large values of λ the same formulas mainly describe the "skidding" of the wheel when the tire loses the adherence to the ground. In Fig. 4 a block scheme of the model involving Pacejka formulas is shown, where $\mathbf{V}_c$ and $\mathbf{F}_c$ are respectively the velocity and the force vectors at the contact point.

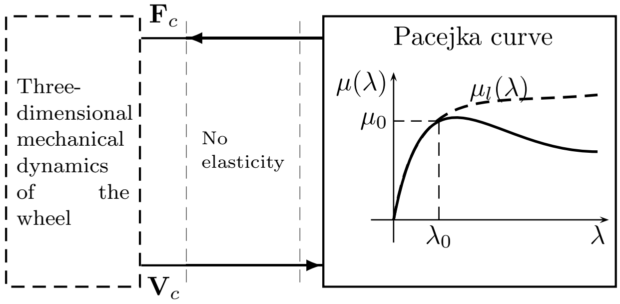


Figure 4. Block scheme of the wheel and the tire-ground interaction using Pacejka formula.

In this paper the Power-Oriented Graphs technique is used for modelling the elastic interaction of the tire with the ground. A three-dimensional energetic model of the tire-ground interaction has been introduced in [6]. The proposed model solves some particular limits of the Pacejka formulas. For example it can be used also when the car or wheel velocities are zero or when the vehicle is at rest on an inclined surface. Moreover, the skidding and the slipping phenomena that in the Pacejka's formulas are mixed, in the proposed model are kept separate so allowing a more direct correspondence of the model with the physical meaning of the described phenomena.

Let us consider the dynamic system shown in Fig. 5 composed by a wheel with an elastic interaction with the ground. In this paper only the longitudinal component of friction is considered, in order to simplify the approach and allow a comparison with the well-known LuGre dynamic friction model [4], [2]. The dynamic model of the

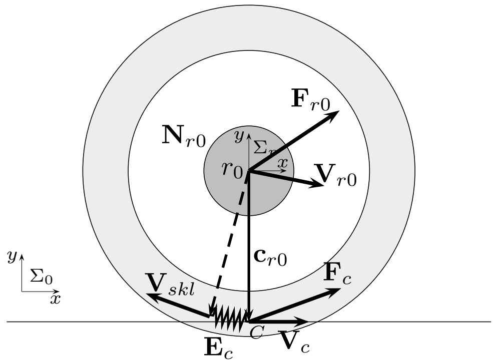


Figure 5. The considered system: a wheel with an elastic interaction with the ground.

considered system described with Power-Oriented Graphs technique is shown in Fig.6, where the wheel dynamics are condensed in the dashed bordered block. This representation is the analog of Fig. 4, highlighting the corresponding power sections. The dynamics of the elastic interactions

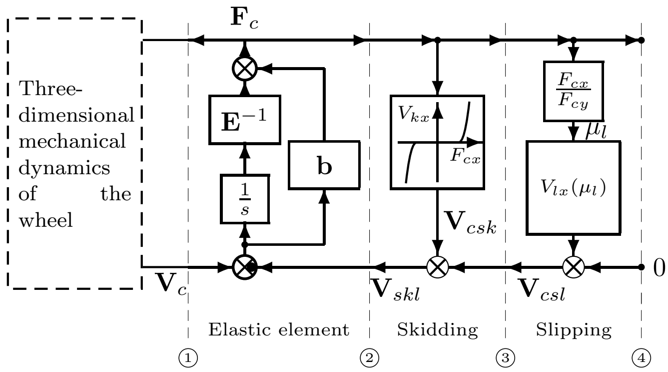


Figure 6. The POG dynamic model of the wheel and the elastic tire-ground interaction.

of the tire with the ground is represented by the POG elaboration block present between sections ① and ② . Matrix $\mathbf{E}^{-1}$ is the two-dimensional stiffness matrix and $\mathbf{b}$ is the two-dimensional damping matrix of the tire-soil contact area (in this case the contact patch is a segment as only the x component is considered):

$$\mathbf{E}^{-1} = \text{diag}\{K_x, K_y\}, \tag{4}$$

$$\mathbf{b} = \text{diag}\{b_x, b_y\}, \tag{5}$$

where K_x, K_y are the translational stiffness coefficients of the tire-ground contact areas and b_x, b_y are the damping coefficients. Note that in (4) and (5) only the particular case of deformations along x and y axis is considered. Matrix $\mathbf{E}^{-1}$ relates the elastic displacement to the force vector $\mathbf{F}_c$ generated in the contact point. The elastic element $\mathbf{E}^{-1}$ located in the contact point is characterized at an end by the rolling velocity $\mathbf{V}_c$ of the tire and at the other end by the sum of the "skidding" velocity $\mathbf{V}_{csk}$ and the "slipping" velocity $\mathbf{V}_{csl}$. Blocks between sections ② and ④ represent the slip and skid phenomena. For details see [6].

The skidding. The POG elaboration block present between power sections ② and ③ describes the "skidding" of the tire on the ground. The tire starts skidding when the contact force exceeds a *skidding threshold*, i.e. when there is a loss of adherence. The skidding velocity $\mathbf{V}_{csk}$ is defined as to have always the same direction of the displacement vector: this means that the skidding is always described as a dissipative phenomenon. For each wheel the skidding velocity is $\mathbf{V}_{csk} = [V_{kx},\, 0]^{\mathrm{T}}$ with

$$V_{kx} = \begin{cases} 0 & \text{if } F_{cx} \leq a \\ K_s (F_{cx} - a)^2 & \text{otherwise} \end{cases}$$

where F_{cx} is the first component of the force vector $\mathbf{F}_c$, K_s is a proper positive coefficient and a is the output of a first order linear filter $\dot{a} = \frac{1}{\tau_{hy}}(a_{hy} - a)$ characterized by the time constant τ_{hy} and by the input $a_{hy} = a_{hy}(F_{cx})$ shown in Fig. 7.1.

The slipping. The POG elaboration block present between power sections ③ and ④ describes the "slipping" of the tire on the ground. The tire slips only when both the contact force F_{cx} and the angular velocity of the wheel ω are not zero. For each wheel the slipping velocity of the contact point is $\mathbf{V}_{csl} = [V_{lx}, 0]$ with:

$$V_{lx} = \min(|\omega\, R_e|, |\,\dot{x}|)\; |\lambda_s(\mu_l)|$$

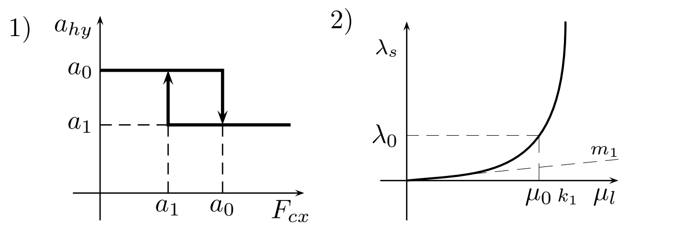


Figure 7. 1) Hysteresis function: the input a_{hy} of the linear filter is a function of F_{cx}; 2) Slipping function $\lambda_s(\mu_l)$ that describes the slip ratio λ_s as a function of the friction coefficient μ_l.

where R_e is the effective rolling radius of the wheel, $\mu_l = \frac{F_{cx}}{F_{cy}}$ is the modulus of the friction coefficient of the contact point and function $\lambda_s(\mu_l)$ describes the slipping behaviour shown in Fig. 7.2, where coefficients m_1 and k_1 describe respectively the slope in the origin and the abscissa of the asymptote of the function.

The main advantages of the energetic approach are that the model is always dissipative, it can be used even in particular operating conditions when other models fail and it keeps separated the two phenomena of skidding and slipping.

The proposed energetic model keeps a direct correspondence with the physical elements of the system, in particular it can be seen as the parallel of a spring and a damper, in series with two damping elements standing for skid and slip, as it is shown in Fig. 8: power sections are shown according to the POG scheme of Fig. 6.

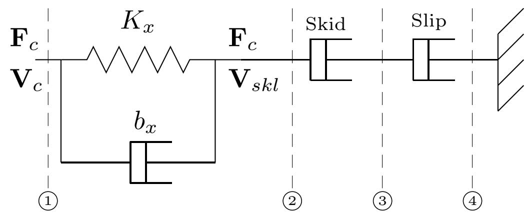


Figure 8. Representation of the physical system corresponding to the energetic model.

4. THE LUGRE MODEL

The LuGre model only refers to the longitudinal friction force, while the other forces are not described. The equations are [5]:

$$\begin{cases} y = \sigma_0 x + \sigma_1 \dot{x} + \alpha_2 u \\ \dot{x} = -\sigma_0 a(u) x + u \\ a(u) = \dfrac{|u|}{(\alpha_0 + \alpha_1 e^{-(u/v_0)^2})} \end{cases} \tag{6}$$

where u is the velocity between the two surfaces in contact, x is the internal friction state and y is the longitudinal friction force. Parameter σ_0 is the normalized rubber longitudinal stiffness, σ_1 is the normalized rubber longitudinal damping, α_2 is the normalized viscous relative damping, α_0 and α_1 are positive coefficients taking into account the Coulomb friction and the Stribeck effect. Function $a(u)$ catches both slip and skid phenomena, it has no unique physical meaning as its shape is a data-fitting based on Pacejka's curves. A POG-like representation of the LuGre model for the longitudinal friction is given in Fig.9.

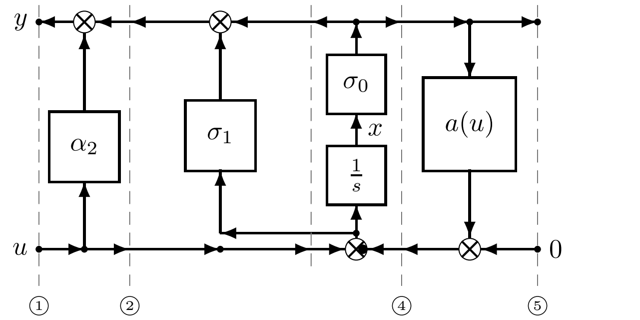


Figure 9. POG-like scheme of the LuGre model of the tire-ground interaction in the longitudinal direction.

4.1 Remarks on dissipativity of the LuGre model

The LuGre friction model (6) is not always dissipative. In [9] they provide necessary and sufficient conditions for passivity of the LuGre friction model, but some remarks about the necessary condition are mandatory.
In order to prove necessity they provide one particular solution for which the passivity is violated. But this does not prove that for all solutions that do not satisfy the condition then the passivity is violated. To prove necessity they should show that if the system is dissipative, then the condition holds. This proof only proves that this model is not dissipative for all conditions and that there is a sufficient condition to make it dissipative. Therefore it seems that this condition is not necessary. In order to prove this it is sufficient to find a solution for which the system is dissipative but the condition is violated and this can be shown in a numerical way. In fact the system can satisfy the dissipative condition even if parameters do not satisfy the claimed necessary and sufficient condition.

Fig. 10 shows the physical correspondent of the LuGre model: α_2 and $a(u)$ are dissipations, σ_0 stands for an elastic element and σ_1 has no physical correspondent. In particular element σ_1 generates a force that has no "reaction", as it acts only on one power section instead of two. In order to maintain the contribute of σ_1 but having

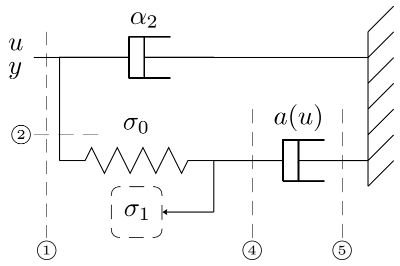


Figure 10. Representation of the physical system corresponding to the LuGre model (6).

a dissipative model, it is possible to modify the model as shown in Fig. 11. The meaning of σ_1 here is a damping element in parallel with the elasticity, like element b_x in the energetic model in Fig. 8. The corresponding equations are:

$$\begin{cases} y = \sigma_0 x + \sigma_1 \dot{x} + \alpha_2 u \\ \dot{x} = -\left[\sigma_0 x + \sigma_1 \dot{x}\right] a(u) + u \\ a(u) = \dfrac{|u|}{(\alpha_0 + \alpha_1 e^{-(u/v_0)^2})} \end{cases} \tag{7}$$

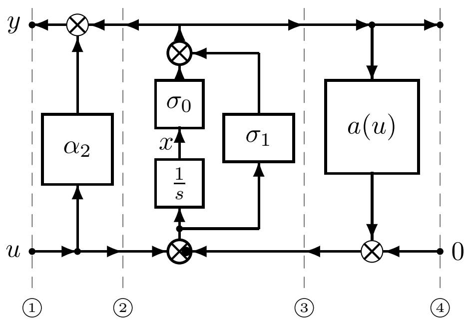


Figure 11. POG scheme of the modified LuGre model.

The modified system (7) is always dissipative when parameters are positive.

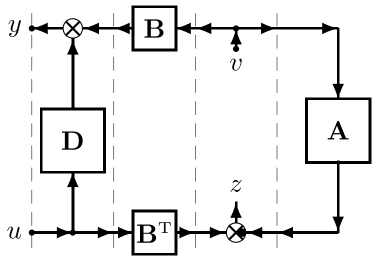


Figure 12. POG scheme of the modified LuGre model without algebraic loop.

The system represented in Fig. 11 contains an algebraic loop involving σ_1 and $a(u)$. This can be solved as it is shown in Fig. 12, where the dynamic element has been disconnected (see the energetic port v-z) and the matrices are: $\mathbf{A} = \frac{a(u)}{1+\sigma_1\, a(u)}$, $\mathbf{B} = \frac{1}{1+\sigma_1\, a(u)}$, $\mathbf{B}^{\mathrm{T}} = \frac{1}{1+\sigma_1\, a(u)}$, $\mathbf{D} = \frac{\sigma_1}{1+\sigma_1\, a(u)}$. The representation of the physical system cor-

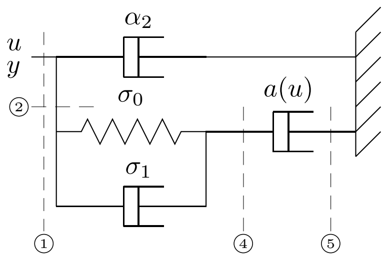


Figure 13. Representation of the physical system corresponding to the modified LuGre model.

responding to the modified LuGre model given by (7) is shown in Fig. 13. Note that this system is more similar to the proposed energetic one (see Fig.6 and Fig.8).

5. SIMULATION

The proposed energetic model and the LuGre model have been implemented in Simulink/SimMechanics environment. A single wheel model is considered and the interaction with the ground is considered with both friction models. The main parameters used in simulation are:...............

6. CONCLUSION

In the paper an energetic dynamic model of the elastic interaction of a tire with the ground has been presented.

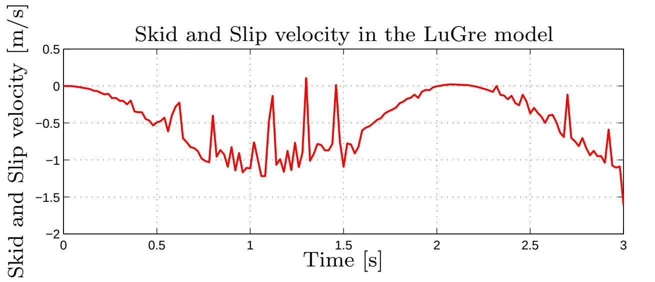


Figure 14. Skid and Slip velocity of the contact point in the LuGre model.

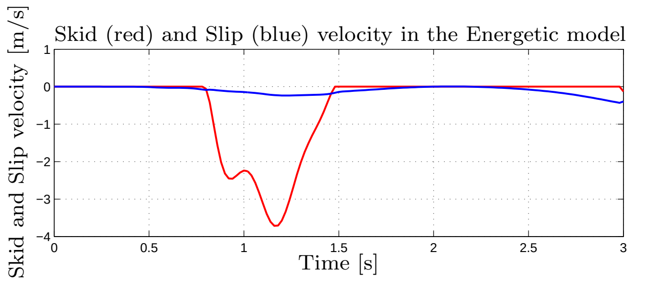


Figure 15. Skid and Slip velocity of the contact point in the Energetic model.

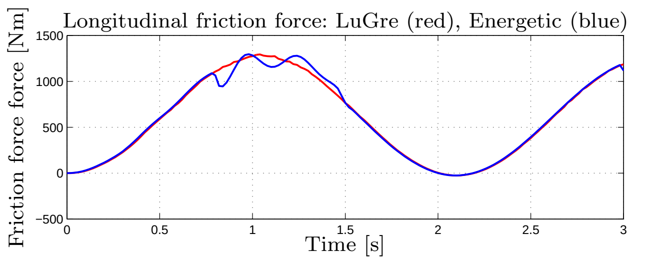


Figure 16. Friction longitudinal force in LuGre (red) and Energetic model (blue).

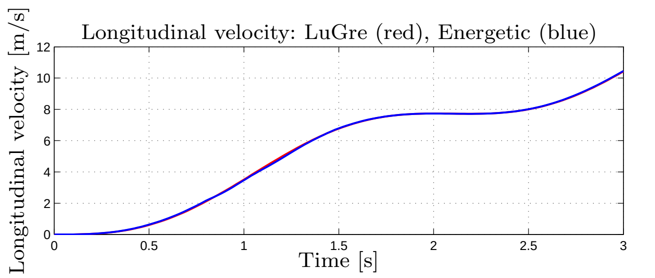


Figure 17. Longitudinal velocity in LuGre (red) and Energetic model (blue).

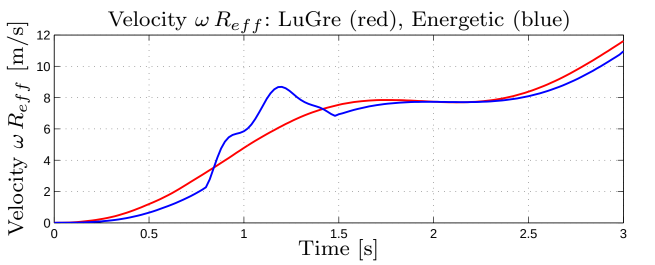


Figure 18. Velocity $\omega\, R_{eff}$ in LuGre (red) and Energetic model (blue).

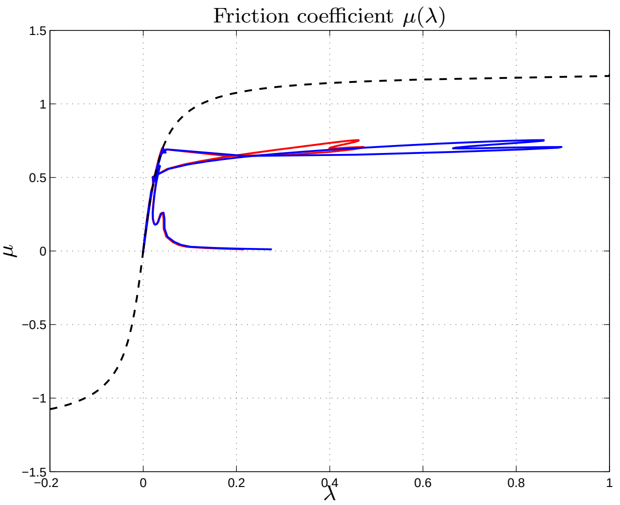


Figure 19. Friction coefficient $\mu(\lambda)$ in the Energetic model.

An important feature of the proposed model is the use of an elastic element for describing the interaction of the tire with the ground. This elastic element, which surely is present in the physical system, is not considered in the Pacejka model. So, the presented model seems to be particularly suitable for the study of all the physical situations in which the presence of the tire elasticity can be important, such as, for example, the design and the tuning of the ABS and ESP control systems. The system has been modelled using the POG graphical technique and then implemented in Simulink environment.

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